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Book Review: *Regular and Stochastic Motion*

Regular and Stochastic Motion. A. J. Lichtenberg and M. A. Lieberman, *Applied Mathematical Sciences*, Vol. 38.
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The field of nonlinear dynamics has grown substantially during the last decade both in the number of practitioners and in the number of results obtained. The activity has been distributed among the arenas of traditional mathematical analysis, the development of computer algorithms and new perturbation techniques, and the uncovering of applications in condensed matter physics, chemical reactions, astronomy, plasma physics, hydrodynamics, meteorology, biology, and so on. There has been concerted effort in the design of high-precision experiments to test the physical implications of many of the generic concepts and conjectures that have evolved in these various areas. Because many of the experiments have yielded positive results it has become advisable for the physics/engineering community at large to become familiar with the new techniques, if not to master them. A number of mathematical monographs on various aspects of the subject have been published fairly recently, but until the present text no authoritative work directed toward the physics/engineering community has appeared.

The present work satisfies the need for a practitioners text by emphasizing physical insight rather than mathematical rigor and presents practical methods for describing the motion of nonlinear oscillator systems. The analyses are based on perturbation theories and the discussions revolve around the limitations of the various guises of perturbation theory. The authors have in large part succeeded in describing where various perturbation theories break down and how computer calculations are used to penetrate the cloud of divergent perturbation series into the realm of chaos. A characteristic of the text is its strong leaning toward applications in plasma physics, which the reader may or may not find appealing. I would have preferred more examples drawn from geophysics and biology to broaden the readership of the book.

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The book is essentially self-contained. The first chapter is an excellent review of the fundamental concepts of classical mechanics containing a smooth transition from the traditional dynamic equations for Hamiltonian systems to the concept of area-preserving mappings. The complexity of the dynamics is introduced in a step-by-step manner, e.g., the pendulum Hamiltonian is introduced as a basic pedagogical tool and its basic structure appears over and over in subsequent chapters. The theme of the book, i.e., regular trajectories and coexisting regions of chaos, is introduced and a brief overview of the entire work is presented. The level of text is that of a physics graduate course in analytic mechanics.

The second chapter is a collection of recent (and some not so recent) techniques for solving nonlinear dynamical problems by perturbing away from known solutions. The presentation stresses the failure of perturbation theory to describe basically nonintegrable systems in which the formal perturbation series diverge. The fundamental problem of resonance, the standard Hamiltonian, and KAM theory are discussed in some detail with many examples drawn from plasma physics.

The third chapter develops the concept of a mapping that can arise from (1) the statement of a problem, (2) the intersection of continuous orbits with a surface of section, or (3) as an approximation to the true dynamic equations. Much of the mathematical jargon is introduced and described in this chapter, e.g., KAM stability, hyperbolic and elliptic fixed points, homoclinic and heteroclinic points, etc. The problem of stability and the transition from regular to chaotic motion is discussed, but in Chapter 4 many pictures showing this transition are presented. In this latter chapter the fact that this is a practitioners work book becomes clear. In the absence of an analytic theory to describe the transition to global stochasticity, five approximation techniques are used each to give a different insight into the transition process.

In Chapter 5 the connection is made between intrinsic chaotic motion in the Hamiltonian systems and the concept of random or stochastic motion as it is traditionally employed in statistical mechanics. In particular the concepts of Liapunov exponents, KS entropy, and diffusion in action space are discussed. The Fokker–Planck equation for the action is constructed from the limits of a map in which the correlation time between successive collisions becomes negligible relative to the characteristic time scales in the system. Chapter 6 addresses the question of the effect of many degrees of freedom on the nonlinear oscillator systems by means of the concepts of Arnold diffusion. A number of simple examples are used to calculate the rate of Arnold diffusion.

In the final chapter an all-too-brief introduction to the evolution of dissipative systems is presented. The notions of strange attractor and fractal dimensions are related by the fractional occupancy of the phase space (minimum of three dimensions) by the trajectory. The cascade of period doubling bifurcations leading to chaotic behavior of the attractor is reviewed and again the Fokker–Planck equation is used to calculate invariant distribution functions. The Fokker–Planck solution is used as a first approximation in finding the invariant distribution on a strange attractor.

In summary this is an excellent graduate level text and is sufficiently up to date to also be useful as a reference work. My major criticism of the book is the limited scope of the examples used. This constraint on the examples facilitates certain of the technical discussions, but I feel that it unnecessarily limited the audience that could make effective use of the text. Thus my criticism is actually a solicitation for more of the same applied in other fields.

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